Math 31 - Homework 1

Due Friday, June 28

Easy

1. Find $\gcd(a, b)$ and express $\gcd(a, b)$ as $ma + nb$ for:

(a) $(116, -84)$

(b) $(85, 65)$

(c) $(72, 26)$

(d) $(72, 25)$

2. Verify that the following elements of $\langle \mathbb{Z}_n, \cdot \rangle$ are invertible, and find their multiplicative inverses.

(a) 4 in $\mathbb{Z}_{15}$

(b) 14 in $\mathbb{Z}_{19}$

3. In each case, determine whether $*$ defines a binary operation on the given set. If not, give reason(s) why $*$ fails to be a binary operation.

(a) $*$ defined on $\mathbb{Z}^+$ by $a * b = a - b$.

(b) $*$ defined on $\mathbb{Z}^+$ by $a * b = a^b$.

(c) $*$ defined on $\mathbb{Z}$ by $a * b = a/b$.

(d) $*$ defined on $\mathbb{R}$ by $a * b = c$, where c is at least 5 more than $a + b$.

4. Determine whether the binary operation $*$ is associative, and state whether it is commutative or not.

(a) $*$ defined on $\mathbb{Z}$ by $a * b = a - b$.

(b) $*$ defined on $\mathbb{Q}$ by $a * b = ab + 1$.

(c) $*$ defined on $\mathbb{Z}^+$ by $a * b = a^b$.

5. [Saracino, Section 1, #1.9] If S is a finite set, then we can define a binary operation on S by writing down all the values of $s_1 * s_2$ in a table. For instance, if $S = \{a, b, c, d\}$, then the following gives a binary operation on S.

*	a	b	c	d
a	a	c	b	d
b	c	a	d	b
c	b	d	a	c
d	d	b	c	a

Here, for $s_1, s_2 \in S$, $s_1 * s_2$ is the element in row s_1 and column s_2. For example, $c * b = d$. Is the above binary operation commutative? Is it associative? (**Note:** The sort of table described in this problem is sometimes called a **Cayley table**.)

6. Compute the Cayley table for $\langle \mathbb{Z}_6, +_6 \rangle$.

Medium

7. Suppose that $*$ is an associative and commutative binary operation on a set S. Show that the subset

$$H = \{a \in S : a * a = a\}$$

of S is closed under $*$. (The elements of H are called **idempotents** for $*$.)